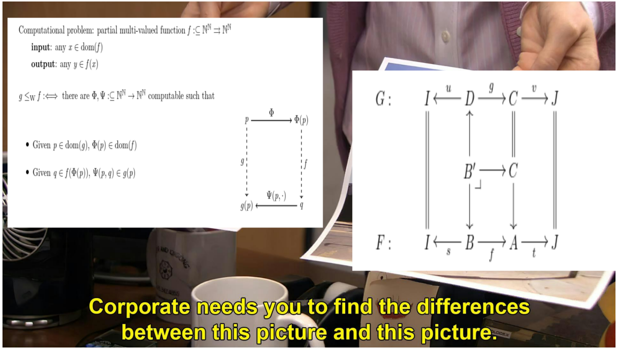
Weihrauch Reducibility as a Lens

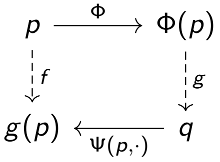
Ian Price (countingishard.org)



Reducibility

Definition (Weihrauch Reducible)

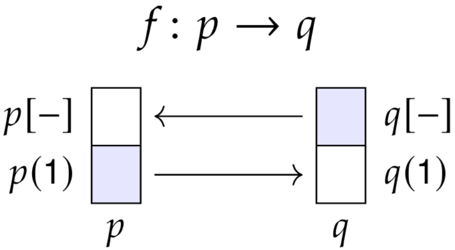
Let X, Y, Z, W be represented spaces and let $f :\subseteq X \rightrightarrows Y$, $g :\subseteq Z \rightrightarrows W$ be partial multi-valued functions.
Then f is *Weihrauch reducible* to g, if there are computable partial functions $\Phi, \Psi :\subseteq \mathbb{N}^{\mathbb{N}} \to \mathbb{N}^{\mathbb{N}}$ such that for all p with $\delta_X(p) \in \mathrm{dom}(f)$, $\delta_Z(\Phi(p)) \in \mathrm{dom}(g)$ and for all q with $\delta_W(q) \in g(\delta_Z(\Phi(p)))$, $\delta_Y(\Psi(p, q)) \in f(\delta_X(p))$.



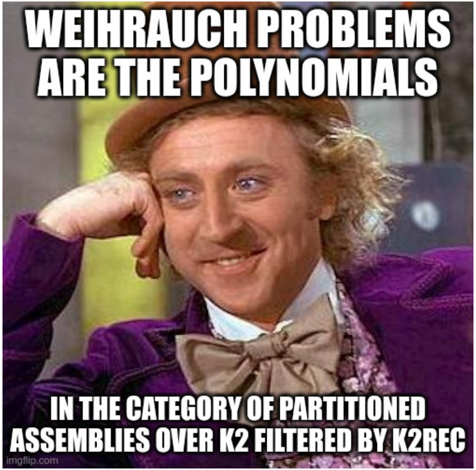
Lenses

Theorem (Niu, Spivak)

Given polynomial functors p and q, a natural transformation (lens) $f : p \to q$ can be identified with a pair $(f_1, f^\#)$ where $f_1 : p(1) \to q(1)$ is a function and $f^\# : q[f_1(-)] \to p[-]$ is a natural transformation (a family of functions $f_i^\# : q[f_1(i)] \to p[i]$ for $i \in p(1)$).

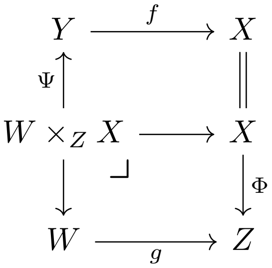


My Advisor Weighs In



Long Story Short

- Sets Bad, Type 2 computability Good
- Pasm($\mathcal{K}_2^{\text{rec}}, \mathcal{K}_2$) is not an LCCC, so can't define Polynomial Functors
- but you can mimic *vertical-cartesian* factorisation
- this is enough



Vertical-Cartesian factorisation

Thank You

Details at countingishard.org