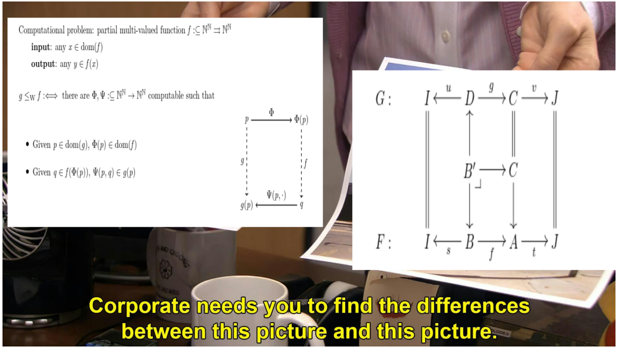
Weihrauch Problems as Containers

Ian Price (countingishard.org) jww Cécilia Pradic



Turing Reducibility: A refresher

- A problem A is reducible to a problem B if there is a turing machine that can solve A given access to an oracle for B
- This is useful for looking at the structure of non-computable logical statements
- LPO: Decide if $w \in \{0, 1\}^{\mathbb{N}}$ is constantly 0
- WKL: Find an infinite path in an infinite binary tree
- Q: Is LPO reducible to WKL, or vice versa? Equivalent? Incomparable?

Weihrauch Reducibility

What if you could only make a single oracle call?

```
def problem(arg):
  x = phi(arg)
  res = oracle(x)
  ans = psi(arg, res)
  return ans
```

$$\begin{tikzcd} \mathrm{arg} \arrow[r, "\Phi"] \arrow[d, dashed, "\mathrm{problem}"] & \Phi(\mathrm{arg}) \arrow[d, dashed, "\mathrm{oracle}"] \\ \mathrm{problem}(\mathrm{arg}) & \mathit{res} \arrow[l, "\Psi(\mathrm{arg},\cdot)"] \end{tikzcd}$$

My lens (container morphism) sense is tingling. . .

My Advisor Weighs In

Containers

- Hopefully nothing I say will contradict Thorsten's mini-lecture (much)
- Containers are (roughly) the category theory equivalent of families of sets $(X_i)_{i \in I}$
- Containers $X \triangleright Y$ are pairs $(X \in \mathbb{C}, Y \in (C/X))$
- morphisms $(X \triangleright Y) \to (Z \triangleright W)$ are pairs $(\Phi : X \to Z, \Psi : (\Phi *)W \to Y \in (\mathbb{C}/X))$

$$\begin{tikzcd}
Y \arrow[r, "f"] & X \arrow[d, equal] \\
W \times_Z X \arrow[u, "\Psi"] \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & X \arrow[d, "\Phi"] \\
W \arrow[r, "g"'] & Z
\end{tikzcd}$$

Where we're at

- Category for Type 2 computability ✓(Pasm($\mathcal{K}_2^{\mathrm{rec}}, \mathcal{K}_2$))
- Weihrauch degrees ✓
- Lattice Operations ✓
- Monoidal Product ✓
- Composition of Containers ✓(Sorta)
- Strong Weihrauch Reducibility ✓(NEW)
- Fixed Points (Soon?)
- Quotient of Degrees
- Parallelisation

Thank you

Blog post at countingishard.org
CiE submission on Arxiv